"Dressing" and Haag's theorem*

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Abstract

It is demonstrated that the "dressed particle" approach to relativistic local quantum field theories does not contradict Haag's theorem. On the contrary, "dressing" is the way to overcome the difficulties revealed by Haag's theorem.

1 Introduction

Two corpuscular interpretations are well-known for relativistic local theories of interacting fields: i.e., those in terms of "bare" and *in-out* particles. The first is the corpuscular interpretation of free fields, though used in the case when interactions between the fields are turned on. Its drawbacks are known. In order to formulate scattering problems correctly one uses *in-out* operators. However, the task of determining them in a given Lagrangean theory coincides, in fact, with the task of diagonalizing of the full Hamiltonian H. Usually, the *in-out* operators are not calculated, but postulated.

In this work, a corpuscular interpretation in terms of "dressed" particles is discussed. A "dressed" particle is to be understood as a particle described by creation-annihilation operators $\alpha^\dagger, \alpha$ with the following properties:

a) The spectrum of indices enumerating $\alpha^\dagger, \alpha$ should be the same as for "bare" operators $a^\dagger, a$. The commutation relations for $\alpha^\dagger, \alpha$ are also canonical:

$$[\alpha_{\mathbf{p}}, \alpha^\dagger_{\mathbf{p}'}] = \delta(\mathbf{p} - \mathbf{p}') \tag{1}$$

The usual Fock representation of (1) is adopted, so that a particle number operator exists, as is necessary for the corpuscular interpretation.

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b) The no-particle vector Ω of this representation ($\alpha_{\mathbf{p}}\Omega = 0$ for all $\mathbf{p}$) coincides with the physical vacuum of the theory (the eigenvector of H with the lowest energy).

c) The one-particle states $\alpha^{\dagger}_{\mathbf{p}}\Omega$ must be also eigenvectors of H.

These fundamental requirements (cf. [1]) are usually augmented by a number of additional ones. For example, the state $\alpha^{\dagger}_{\mathbf{p}}\Omega$ should be an eigenstate of the total momentum, and should possess definite quantum numbers, like charge, parity, etc. (see [2] and Sect. 3.1 in [3]). However, for the present work only properties a), b) are significant.

There exist other definitions of "dressing". See the end of section 2. Note that *in-out* operators satisfy requirements a), b), c), but in addition they also have the property of stationarity for all states of the form $\alpha^{\dagger}_{\mathbf{p}_1}\ldots\alpha^{\dagger}_{\mathbf{p}_n}\Omega$. For more about the connection between the *in-out* and "dressed" operators see section 2 below. Section 2 is mainly devoted to demonstrating non-locality of the "dressed field". This property of "dressing" is used in a very substantial way in section 3, where the main statement of this work is proved: that one cannot reject the possibility of a "dressed" corpuscular interpretation of relativistic local field theory on the basis of Haag's theorem. On the contrary, "dressing" is the way to overcome the difficulties revealed by Haag's theorem.

2 "Dressing" and non-locality

Here we present a simplified version of the formal "dressing" procedure, due to Faddeev [4]. This procedure is applicable to any relativistic local field theory.

Let us assume, for definiteness, that the interaction Hamiltonian density is the product of three field operators (quantum electrodynamics $\overline{\psi}\gamma_\mu\psi A_\mu$, Yukawa interaction $\overline{\psi}\psi\phi$, etc.). The annihilation operators of "bare" particles (electrons, photons, nucleons, mesons) are denoted by $a_{\mathbf{p}}$. The corresponding no-particle vector Ω_0 ($a_{\mathbf{p}}\Omega_0 = 0$ for all $\mathbf{p}$) is an eigenvector of the free part H_0 of the Hamiltonian. However, it is not an eigenvector of the total Hamiltonian H, because there are interaction terms that contain products of only (three) creation operators. The interaction terms containing products of two creation operators and one annihilation operator (we call them terms of type (2,1)) do not allow the one-particle states $a^{\dagger}_{\mathbf{p}}\Omega_0$ to be eigenvectors of H. Other interaction terms (of type (0,3) and (1,2)) are Hermitian conjugate to those just mentioned.

Instead of $a_{\mathbf{p}}$, let us introduce new operators $\alpha_{\mathbf{p}}$, which are related to $a_{\mathbf{p}}$ by a formally unitary transformation

$$a_{\mathbf{p}} = W\alpha_{\mathbf{p}}W^{\dagger},\ a^{\dagger}_{\mathbf{p}} = W\alpha^{\dagger}_{\mathbf{p}}W^{\dagger} \tag{2}$$

(so that properties a) from the Introduction are satisfied). One simplest example: $W = \exp\{\frac{1}{2}\int d^3p\chi(|\mathbf{p}|)[\alpha_{\mathbf{p}}\alpha_{-\mathbf{p}} - \alpha^\dagger_{\mathbf{p}}\alpha^\dagger_{-\mathbf{p}}]\}$ corresponds to the linear transformation

$$a_{\mathbf{p}} = \cosh\chi(|\mathbf{p}|)\alpha_{\mathbf{p}} + \sinh\chi(|\mathbf{p}|)\alpha^\dagger_{-\mathbf{p}} \tag{3}$$

Let $H(a^\dagger, a)$ denote the full Hamiltonian expressed in terms of "bare" operators $a^\dagger, a$. If in this expression we transform from $a^\dagger, a$ to $\alpha^\dagger, \alpha$ we obtain the full Hamiltonian as a function of $\alpha^\dagger, \alpha$:

$$H(a^\dagger, a) = H(W\alpha^\dagger W^\dagger, W\alpha W^\dagger) = WH(\alpha^\dagger, \alpha)W^\dagger = K(\alpha^\dagger, \alpha) \tag{4}$$

(where we used formulas of the type $f(W\alpha W^\dagger) = Wf(\alpha)W^\dagger$). Now we need to construct W such that the full transformed Hamiltonian $K(\alpha^\dagger, \alpha)$ would not contain "bad" terms of type (3,0), (2,1), and generally of types $(m, 0)$ and $(m, 1)$ with $m \geq 2$. It is precisely these "bad" terms which prevent the no-particle vector Ω and the vectors $\alpha^\dagger_{\mathbf{p}}\Omega$ from being eigenvectors of $K(\alpha^\dagger, \alpha)$.[1]

The operator W is constructed in the following way. Let $H(a^\dagger, a) = H_0(a^\dagger, a) + \lambda V(a^\dagger, a)$, where λ is a small coupling constant. Then we represent W in the form $\exp R(\alpha^\dagger, \alpha)$, where the anti-Hermitian operator R has the form $R = \sum_n \lambda^n R_n$, $R^\dagger_n = -R_n$.[2]

To determine $K(\alpha^\dagger, \alpha) = WH(\alpha^\dagger, \alpha)W^\dagger$ we use the formula

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2}[A, [A, B]] + \frac{1}{3!}[A, [A, [A, B]]] + \ldots$$

to obtain a power series in λ

$$K(\alpha^\dagger, \alpha) = K_0(\alpha^\dagger, \alpha) + \lambda K_1(\alpha^\dagger, \alpha) + \lambda^2 K_2(\alpha^\dagger, \alpha) + \ldots; \tag{5}$$

$$K_1 = [R_1, H_0] + V;\ K_2 = [R_2, H_0] + [R_1, V] + \frac{1}{2}[R_1, [R_1, H_0]] + \ldots \tag{6}$$

K_1 contains interaction terms V. All are "bad" in the case of the three-operator interaction. By an appropriate choice of R_1 we can make K_1 zero. To do this, we choose R_1 to be a three-operator expression of the same structure as V, but with other coefficient functions. Then $[R_1, H_0]$ is also a three-operator expression, which can be made equal to $-V$ by an appropriate choice of these coefficient functions. (Note that the masses of particles should be such that the decay of one particle into two is impossible).

[1]Note that terms of type (1,1) are allowed. The free part of K is composed of them.

[2]This representation simplifies the procedure suggested in [4].

After finding R_1, we can calculate all terms in K_2 except $[R_2, H_0]$, see (6). There are "bad" terms among them. To find them, we perform normal ordering of terms $[R_1, V]$ and $[R_1, [R_1, H_0]] = -[R_1, V]$. (I.e: we move all creation operators to the left of any annihilation operators by using the commutation relations (1)). Thus we obtain "bad" terms of types (2,0), (4,0), (3,1). If we take R_2 as a superposition of terms of the same types, then the corresponding coefficient functions in R_2 can be chosen such that $[R_2, H_0]$ compensates "bad" terms from $[R_1, V]$. Similarly, one can delete "bad" terms from K_n with any n [4].

Then in K_2, K_3, etc, only "good" terms are left, e.g: of type $\alpha^\dagger\alpha^\dagger\alpha\alpha$. They describe interactions that lead to scattering and more complicated reactions. Let us now demonstrate that these interactions are non-local.

We formally introduce the Heisenberg operator of the "dressed field"

$$A(\mathbf{x}, t) = (2\pi)^{-3/2} \int d^3p \frac{1}{\sqrt{2E_\mathbf{p}}} [e^{-iE_\mathbf{p}t + i\mathbf{p}\mathbf{x}} \alpha(\mathbf{p}, t) + e^{iE_\mathbf{p}t - i\mathbf{p}\mathbf{x}} \alpha^\dagger(\mathbf{p}, t)] \tag{7}$$

which is built in the usual manner from the Heisenberg "dressed" creation-annihilation operators $\alpha(\mathbf{p}, t) = \exp(iHt)\alpha_\mathbf{p}\exp(-iHt)$. As with $\alpha^\dagger, \alpha$, the symbol A represents a set of operators for all fields in the theory under consideration.

If the four-operator part of the interaction were local, for example of type $gA^4(x)$, then in addition to $\alpha^\dagger\alpha^\dagger\alpha\alpha$ the Hamiltonian K would necessarily also contain terms of the type $\alpha^\dagger\alpha^\dagger\alpha^\dagger\alpha^\dagger$ and $\alpha^\dagger\alpha^\dagger\alpha^\dagger\alpha$. However, such "bad" terms were removed from K.

The non-locality referred to above means this: if $A(x) = A(\mathbf{x}, t)$ satisfies an equation of the type $(\Box + m^2)A(x) = J(x)$, then the current $J(x)$ is non-local in the sense that

$$[J(x), A(y)] \neq 0 \ when\ (\mathbf{x} - \mathbf{y})^2 > (x_0 - y_0)^2 \tag{8}$$

We conclude this section with three remarks.

1. For local interaction, Faddeev's procedure leads to divergences. For example, normal ordering of $[R_1, V]$ creates terms of type $\int d^3p\Delta(\mathbf{p})\alpha^\dagger\alpha$, where $\Delta(\mathbf{p})$ is given by a divergent integral. These terms are corrections to the free part of the Hamiltonian (of order λ^2), and $\Delta(\mathbf{p})$ is a correction to the energy $E_\mathbf{p} = \sqrt{\mathbf{p}^2 + m^2}$. Therefore, it is necessary to introduce a momentum cutoff and add renormalization counterterms to the original interaction.

   However, even with these improvements, the expression $\exp R(\alpha^\dagger\alpha)$ is not an operator, as it fails to map vectors of the Hilbert space in the Fock representation of operators α to vectors in the same space. It is known, for instance, that even the simplest W corresponding to the transformation (3) fails in this respect.

(See [5] page 19 and [6], §4). However, expressions of this kind can be given a mathematical meaning by adopting the algebraic point of view presented in [7] and [2].

2. The "dressing" procedure described above enables us to discuss the question of the connection between the "dressed" and *in-out* operators raised in [1].

   The procedure of finding *in-out* operators, which is very similar to the Faddeev's procedure, was suggested simultaneously by Weidlich [7]. It consists of deleting *all* the interaction terms in K_n, not just "bad" ones, so that in terms of *in* operators the full Hamiltonian H must obtain the free form. (It is presumed that this operator does not have bound states). This is equivalent to finding all eigenstates of H, see also [8]. Meanwhile "dressing" is equivalent to the finding only the first few eigenstates of H (the vacuum vector and one-particle states). The issue of bound states requires further investigation (now in terms of "dressed" operators). If there are bound states then the spectrum of indices of the "dressed" operators differs from the spectrum of indices of the *in-out* operators. We note that Heisenberg's "dressed" operators converge strongly, i.e., with respect to the norm, to the *in-out* operators [9, 10].

3. Note that in the literature, the term "dressing" is often applied to transformations which differ from the Faddeev's $W = \exp R$ described above, so that some of the conditions a), b), c) (see Introduction) do not hold. For example, the exponent may include only some terms from the series $\sum_n \lambda^n R_n$, so that W can even be formally non-unitary [11, 12, 13].

3 "Dressing" and Haag's theorem

The procedure based on perturbation theory described above for finding "dressed" operators was formal (since questions of convergence were not discussed). Therefore, it is important to consider objections against the "dressing" approach based on Haag's theorem and other similar theorems [14, 15].

Haag's theorem exists in its original form (see §4 in [5], §6 in [16], and [17]) and in the form of Hall-Wightman, which contains a large number of assumptions (see books [18, 19]).

Let us first demonstrate that "dressing" is one method for overcoming the difficulty raised by the original Haag theorem. To do that, we present a proof of this theorem in the framework of the Lagrangean formalism, which is somewhat different from the proof in [5, 16, 17]. We start with the proof of the following lemma.

Lemma. Suppose we have a Euclidean (i.e., translational and rotational) invariant field theory written in terms of creation-annihilation operators $a^\dagger_{\mathbf{p}}, a_{\mathbf{p}}$. Suppose also

that a Fock representation of these operators in the Hilbert space $\mathcal{H}_0$ with the no-particle vector Ω_0 is given. Then, in $\mathcal{H}_0$ there is a unique normalizable eigenstate of the total momentum operator $\mathbf{P}$, and this state coincides with Ω_0.

The proof consists in analyzing all eigenstates of $\mathbf{P}$. It is known that for any interaction the operator $\mathbf{P}$ has the free form $P_j = \int d^3p p_j a^\dagger_{\mathbf{p}} a_{\mathbf{p}}$.[3] $\mathbf{P}$ is one of the generators of the Euclidean subgroup, so its spectrum should be continuous. Any vector from $\mathcal{H}_0$ can be expanded in the basis $\Omega_0, a^\dagger_{\mathbf{p}}\Omega_0, \ldots, a^\dagger_{\mathbf{p}_1} \ldots a^\dagger_{\mathbf{p}_0}\Omega_0, \ldots$. All of them are eigenvectors of $\mathbf{P}$, but only the vector Ω_0 is normalizable. All other vectors are non-normalizable. Their arbitrary superposition

$$\sum_{n=1}^{\infty} \int d^3p_1 \ldots d^3p_n F(\mathbf{p}_1, \ldots, \mathbf{p}_n) a^\dagger_{\mathbf{p}_1} \ldots a^\dagger_{\mathbf{p}_n} \Omega_0 \qquad (9)$$

is non-normalizable as well, if it is an eigenvector of $\mathbf{P}$. Indeed, (9) is such an eigenvector if F contains a delta-function of the type $\delta(\mathbf{p}_1 + \mathbf{p}_2 + \ldots + \mathbf{p}_n - \mathbf{P})$, but then $\int d^3p_1 \ldots d^3p_n |F(\mathbf{p}_1, \ldots, \mathbf{p}_n)|^2$ diverges.

Remark regarding the uniqueness of the normalized eigenvector of $\mathbf{P}$: Let us write the theory in terms of other creation-annihilation operators $\alpha^\dagger_{\mathbf{p}}, \alpha_{\mathbf{p}}$, such that $\mathbf{P}$ preserves its form $P_j = \int d^3p p_j \alpha^\dagger_{\mathbf{p}} \alpha_{\mathbf{p}}$ [2]. This can be achieved, for example, when the α are related to the a by the transformation (3). The no-particle vector Ω' (for which $\alpha_{\mathbf{p}}\Omega' = 0$) is also a non-normalizable eigenvector of $\mathbf{P}$. For the transformation (3) it can be shown that Ω' does not belong to the Hilbert space constructed cyclically from the vector Ω_0 ([5], page 19).

Theorem. Suppose that conditions of the Lemma are satisfied and there is a unique normalizable eigenstate of the full Hamiltonian H with lowest energy, i.e., the vacuum vector Ω. Then Ω must coincide with Ω_0.

Proof. Since $[H, P_j] = 0$, Ω must be a common eigenvector of H and $\mathbf{P}$. However there is only one normalizable eigenstate of $\mathbf{P}$ in $\mathcal{H}_0$, and this eigenstate coincides with Ω_0. Thus $\Omega = \Omega_0$.

In fact, in all local theories Ω does not coincide with the no-particle vector of "bare" creation-annihilation operators which diagonalize H_0. This means such theories violate some assumptions of the theorem. The usual conclusion [16] is to reject the Fock representation for "bare" operators and to use instead a some "strange" representation without a particle number operator, see, for example §18.3 in [21]. It follows from our **Remark** to the Lemma that the theorem does not forbid the Fock representation

[3] We have in mind "instant form" theories [20], where time is a parameter (the states are given at a fixed time instant).

for operators whose no-particle vector coincides with the vacuum Ω. The "dressed" operators are exactly of this kind.

Let us now discuss the statement of O. Greenberg [14]: the Heisenberg "dressed" operators $\alpha(\mathbf{p},t), \alpha^\dagger(\mathbf{p},t)$, which describe a relativistic field theory and realize the Fock representation of the equal-time commutation relations $[\alpha(\mathbf{p},t), \alpha^\dagger(\mathbf{p}',t)] = \delta(\mathbf{p}-\mathbf{p}')$ must obey the free equation of motion. Greenberg based his derivation on Haag's theorem in Hall-Wightman form. Following [18, 19] we formulate the theorem for our purposes in the following manner.

Suppose we have two field theories. One is a free theory described by a set of free fields $A_0(x)$ acting in the Hilbert space $\mathcal{H}_0$. The other is described by an irreducible set of fields $A(x)$. Further, let us assume that the following conditions are satisfied:

1) $A(x)$ is an operator in $\mathcal{H}$ which carries a unitary representation of translations and rotations

$$U(\mathbf{a},R)A(x)U^\dagger(\mathbf{a},R) = A(Rx+\mathbf{a}) \tag{10}$$

and

1') Lorentz transformations

$$U(\Lambda)A(x)U^\dagger(\Lambda) = A(\Lambda x) \tag{11}$$

(these relationships are written for the particular case of a scalar field).

2) There is a unique invariant state $U\Omega = \Omega$ in $\mathcal{H}$.

3) There exists a unitary operator V, from $\mathcal{H}_0$ to $\mathcal{H}$, such that at a time instant t we have

$$A(\mathbf{x},t) = V(t)A_0(\mathbf{x},t)V^\dagger(t) \tag{12}$$

4) The spectrum of energies is bounded from below.

Then $A(x)$ is a free field.

As a field operator describing the interacting theory we take the "dressed" field operator [8]. Greenberg noticed that the unitary equivalence 3) of the fields $A(x)$ and $A_0(x)$ results from the requirement a) to the "dressed" operators (see Introduction).

Indeed, let us expand $A_0(x)$ in the usual manner using the operators $a_0(\mathbf{p}), a_0^\dagger(\mathbf{p})$, cf. (7). Let $a_0, a_0^\dagger$ also realize a Fock representation of the canonical commutation relations with the no-particle vector $\Omega_0 \in \mathcal{H}_0$. (Note that the choice of one or another representation for the auxiliary operator $A_0(x)$ is under our control). Then, according to a known theorem, the operators $\alpha(\mathbf{p}, t), \alpha^\dagger(\mathbf{p}, t)$ must be connected to $a_0(\mathbf{p}), a_0^\dagger(\mathbf{p})$ by a unitary transformation[4] $\alpha(\mathbf{p}, t) = V(t) a_0^\dagger(\mathbf{p}) V^\dagger(t)$, see [22] and §1.6 in [6]. Therefore, the same transformation connects $A(x)$ and $A_0(x)$.

Assuming that the other conditions of Haag's theorem are satisfied, Greenberg concluded that the "dressed field" $A(x)$ must be free. We show that in this situation condition 1') of the theorem is not satisfied, and therefore such a conclusion is wrong.

We first demonstrate that the local commutation relation

$$[A(\mathbf{x}, x_0), A(\mathbf{y}, y_0)] = 0 \tag{13}$$

is invalid for *all* $(\mathbf{x}, x_0)$ separated by a space-like interval from $(\mathbf{y}, y_0)$. Indeed, if (13) were true, then by applying the operator $(\nabla_x^2 + m^2)$ to (13) we would obtain the relationship $[J(\mathbf{x}, x_0), A(\mathbf{y}, y_0)] = 0$ when $(\mathbf{x}, x_0) \approx (\mathbf{y}, y_0)$. This contradicts the established non-locality of the current J corresponding to the operator A, see (8). For another proof of the impossibility of (13) see [10].

Already, the non-locality of $A(\mathbf{x}, t)$ means that Haag's theorem in this situation cannot be proved using the Jost-Schroer theorem (theorem 4-15 in [18]). In Greenberg's proof, the locality condition for the field is not used (we have not included this condition in the assumptions of Haag's theorem) [14].

We now demonstrate that assuming the validity of 1') for $A(\mathbf{x}, t)$ leads to a contradiction. Note that the non-local field $A(\mathbf{x}, t)$ is such that (13) is valid for *some* $(\mathbf{x}, x_0) \approx (\mathbf{y}, y_0)$. In particular, when $x_0 = y_0 = t$ we have

$$[A(\mathbf{x}, t), A(\mathbf{y}, t)] = 0 \tag{14}$$

This follows from the equal-time commutation relations $[\alpha(\mathbf{p}, t), \alpha^\dagger(\mathbf{p}', t)] = \delta(\mathbf{p} - \mathbf{p}')$ for the Heisenberg "dressed" operators $\alpha(\mathbf{p}, t) = \exp(iHt)\alpha_{\mathbf{p}} \exp(-iHt)$ and from the expansion (7). If the condition 1') were true, then (14) implies the validity of (13) for all $(\mathbf{x}, x_0) \approx (\mathbf{y}, y_0)$, which is impossible.

In fact, from theorems proved in [15, 23], an analogous conclusion follows about the "dressed field": only a non-local "dressed" field can be non-free.

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[4]A transformation is called unitary if it preserves the norm and maps $\mathcal{H}_0$ to the entire space $\mathcal{H}$.

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